The Nature of Propositions

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Philosophy of Language » Lecture 5

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Referentialism and Sentence Meanings: Arguments for Propositions

The Meanings of Sentences

› We have stipulated that the meaning of a sentence is to be called a **proposition**.
  » We say, when sentence *S* means the proposition that *p*, that *S* **expresses** *p*.
  » Because English can talk about its own sentences, this is apt to sound almost trivial – e.g., '*Sylvester eats fish* expresses the proposition that Sylvester eats fish'. But compare: '*Sylvester isst Fisch* expresses the proposition that Sylvester eats fish', or even '*Sylvester isst Fisch* expresses the same proposition as *Sylvester eats fish*'.
› Recall also our working hypothesis of **compositionality**: that the meaning of a grammatically correct phrase is determined by the meanings of its constituents and their syntax, the way the constituents are put together.
› A sentence is a grammatically correct phrase, so its meaning is determined compositionally.

Internalist sentence meanings

› The primary function of an assertion of a **declarative sentence** is to say something about how things are.

› Since **the world is how a sentence says it is iff that sentence is true**, what a declarative sentence says must be capable of **truth** or **falsity**.

  » It may not be possible for us to know or figure out which – truth is distinct from verifiability.

› For an internalist, who thinks the meanings of individual words are concepts in a speaker's head, it is natural to take the meaning of a sentence to be likewise a concept:

> an internal mental structure arrived at in a compositional fashion from the meanings of the words in the sentence and their syntactic arrangement. (Elbourne 2011: 43)

› Given the above, any concept which could be the meaning of a sentence must be one that applies to a scenario (or to 'reality as whole'?) iff the sentence is true.

  » So while *chair* expresses a concept that classifies things, *there is a chair* expresses a concept that classifies **scenarios**.

An Argument for Propositions

Consider the *proposition sentences*:

(1) John said something.
(2) What John said was true.
(3) Though what John said was true, it would have been false had things gone differently.
(4) What John said is what Mary believed.

… the proposition sentences seem to jointly entail:

(5) There is something that John said, and which was true, and which could have been false, and which Mary believed.

… If (5) is true, then there are things which are said and believed, which are the bearers of truth values and have modal properties like being possibly true. So if (5) is true, there are propositions. (Speaks 2014: 10–11)

An Argument for Referentialism about Propositions?

›   This argument takes the **logical form** of (5) at face value: as an existential claim, *there exists a thing which ....*

   »   Elbourne (2011: 44–46) discusses a similar argument offered by Schiffer (2003: 12–14).

›   But while this gives us propositions, it doesn't seem to give us something that fits well with the idea that a sentence meaning is a concept.

›   If what John said (the proposition he expressed) stems from John's belief, then **what John believed** is identical to **what Mary believed**.

›   They are then related by belief to the same proposition; so that proposition would seem not to be a concept in John or Mary's head.

›   Propositions, as supported by this argument, are 'the sharable objects of the attitudes and the primary bearers of truth and falsity' (McGrath and Frank 2024).

Equivocation on *something*?

> … *There is something they both believe* is ambiguous. On [the orthodox] way of reading it, the sentence does indeed assert the existence of an object to which both [John] and [Mary] stand in the belief relation. An internalist will say that on that reading the sentence is simply false…. On another reading, the sentence … would be claiming … that [John]'s belief and [Mary]'s belief are qualitatively identical. We can compare a sentence like *There is something they both own*. This could mean there is a particular concrete object of which [John] and [Mary] are joint owners; but alternatively it could mean that there is a kind of object such that [John] and [Mary] both own objects of that kind. To an assertion that [John] and [Mary]'s possessions are completely dissimilar, it would be possible to reply, 'No, there is something they both own—a house'. This would not necessarily imply that they own the same house. (Elbourne 2011: 45-46)

Does *Something* Really Work That Way?

› Elbourne is arguing that *There is something they both believe* is (roughly) synonymous with *They both believe the same thing*, and so should exhibit **ambiguity** due to the presence of *same* in its covert form – ambiguity between **qualitative** and **numerical** identity.

  » This will help with a reinterpretation of (5); but what about (4) which involves neither *same* nor *something*? Elbourne's suggestion would have to be that *is* sometimes means 'is similar to'.

1. It is arguably nothing more than a pun or play or words to use *there is something they both* Φ to mean *they both* Φ *the same kind of thing*.

  » Witness: *There is something they both hate: themselves*.

2. It also seems the **indefinite** *a house* is significant; it is harder to get Elbourne's reading with the definite: *There is something they both own – the house*; compare *There is something they both accept: the thing John believes*.

3. Think about **explanation**. Suppose two people fall ill simultaneously, and I ask whether they ate something. If you reply, *There is something they both ate*, because they each ate a burger at separate restaurants across town, you are **not** giving the explanation sought.

The Anaphora Argument

› Let's consider another example:

   (6) Harold believes that there is life on Venus, and Fiona believes it too.

› Is it possible to interpret the occurrence of *it* in this sentence as denoting something only **qualitatively similar** to the object of Harold's belief?

› I don't believe so. *It* is **parasitic** upon *that there is life on Venus*.

   » A case of **anaphora**, where the pronoun *it* refers back to some previously introduced linguistic material; in this case, the only candidate is the object of Harold's belief.

› Compare Elbourne's parallel case:

   (7) Harold owns a house, and Fiona owns it too.

› There is no way to interpret this example except as saying that Harold and Fiona are joint owners; similarly with (6). But (6) is true: so there are abstract non-internalist propositions.

The Relational Analysis of Attitudes

› The referentialist offers a simple and elegant account of the attitudes:

The Relational Analysis of Attitude Ascriptions An attitude ascription '*S V*s that *p*' is true iff '*S*' designates a person who stands in the attitude relation expressed by '*V*' to the proposition designated by 'that *p*'…. (McGrath and Frank 2024: §3.1; see also Soames 1987: 48)

› The Relational Analysis can easily explain the validity of this and similar argument forms: the argument on the left has the logical form on the right. Harder for internalists to explain why the argument is good.

Charles believes everything
Thomas says.
Thomas says that cats purr.

$\forall x$(Thomas says that $x \rightarrow$
Charles believes that x).
Thomas says that p.

―――――――――

―――――――――

So, Charles believes that cats purr.

So, Charles believes that p.

Possible Worlds and Propositions

Referentialist sentence meanings

› These arguments for referentialism about propositions still don't answer the question: what are they?

› Two candidate accounts have been popular in the literature:

1. **Structured (Russellian) propositions**. The proposition expressed by *S* is a complex structured mathematical entity, the constituents of which are the meanings of the constituents of *S*. (This picks up the internalist idea that the concept associated with the meaning of a sentence should be a complex concept with further concepts as constituents.)

   » We met this idea in lecture 2, where we said that the meaning of *grass is green* might be the structured proposition ⟨grass, greenness⟩.

2. **Unstructured propositions**. The proposition expressed by *S* is a set of possible situations – those in which *S* is true. (It parallels the internalist idea that there is a collection of scenarios to which the meaning applies.)

Ecumenical Intensions

› What is the extension of a sentence? To what does a sentence 'refer'?

> Every declarative sentence concerned with the reference of its words is there-fore to be regarded as a proper name, and its reference, if it has one, is either the True or the False. (Frege 1892: 29)

› Given this, the **intension of a sentence is a function from possible situations to truth values** (because an intension in general is a function from possible situations to extensions).

› But since there are only two truth values, the intension of a sentence is equivalently a set of possible situations: writing $[\![S]\!]$ for the intension of S, $[\![S]\!] = \{w : S$ is true in $w\}$.

› So an intension just is an unstructured proposition!

› Everyone – internalists and all sorts of referentialists – offers a conception of a proposition on which a proposition **determines** an intension.

› And so most formal semanticists accept that even if they aren't exactly sentence meanings, intensions do a decent job at **modelling** interesting features of sentence meaning (Elbourne 2011: 46).

The truth conditional hypothesis

› So we can start by adopting – for the sake of exploration – the suggestion that a proposition just is – or can be satisfactorily represented as – an unstructured proposition, or intension.

› We'll see how far we can get with the idea that the meaning of a sentence is given by the possible conditions under which it is true – its **truth conditions**.

› We **need not** take this fully literally, though some very distinguished philosophers have:

> a proposition is a set of possible worlds. A proposition is said to *hold* at a world, or to be *true at* a world. The proposition is the same thing as the property of being a world where that Proposition holds; and that is the same thing as the set of worlds where that proposition holds. A proposition holds at just those worlds that are members of it. (Lewis 1986: 53–54)

> a proposition is a function from possible worlds into truth-values. (Stalnaker 1984: 2)

Stalnaker's Classification Argument

[T]he primary objects of attitudes are … alternative possible states of the world. When a person wants a proposition to be true, it is because he has a positive attitude towards certain concrete realizations of that proposition. Propositions … are simply ways of distinguishing between the elements of the relevant range of alternative possibilities – ways that are useful for characterizing and expressing an agent's attitudes toward those possibilities. To *understand* a proposition – to know the content of a statement or thought – is to have the capacity to divide the relevant alternatives the right way. … To *distinguish* two propositions is to conceive of a possible situation in which one is true and the other false. (Stalnaker 1984: 4–5)

According to the conception of content that lies behind the possible worlds analysis of propositions and propositional attitudes, content requires contingency. To learn something, to acquire information, is to rule out possibilities. To understand the information conveyed in a communication is to know what possibilities would be excluded by its truth. (Stalnaker 1984: 85)

The Proposition Role

› Propositions were characterised as 'the shareable objects of the attitudes and the primary bearers of truth and falsity' (McGrath and Frank 2024).

› Stalnaker's argument, in effect, is that sets of possible worlds can play this role, because of their central role in structuring **inquiry** and **belief**.

› They certainly can explain sentential truth/falsity: a sentence S is true at a situation w just in case $w \in \llbracket S \rrbracket$.

› And Stalnaker thinks that the attitudes can be understood as relations to such sets: to **believe** P is to represent the world as among a certain set of p-favouring possibilities; to **learn** Q is to exclude the $\neg Q$ possibilities among those compatible with what you previously knew; to **understand** R is to be able to discriminate what it would take for R to be true among the possibilities one considers; and so on.

Possible Worlds

Possibilities and Possible Worlds

› We've already mentioned 'possible worlds' a number of times – when characterising intensions, when thinking about Kripke's notion of a rigid designator, etc.

› But what are these things? Lewis offers the most radical, and yet most straightforward story:

> When I profess realism about possible worlds, I mean to be taken literally. Possible worlds are what they are, and not some other thing. If asked what sort of thing they are, I cannot give the kind of reply my questioner probably expects: that is, a proposal to reduce possible worlds to something else.
>
> I can only ask him to admit that he knows what sort of thing our actual world is, and then explain that possible worlds are more things of that sort, differing not in kind but only in what goes on at them. (Lewis 1973: 85; see also Lewis 1986)

Lewis' Argument for (Concrete) Modal Realism

I believe there are possible worlds other than the one we happen to inhabit. If an argument is wanted, it is this: It is uncontroversially true that things might have been otherwise than they are. I believe, and so do you, that things could have been different in countless ways. But what does this mean? Ordinary language permits the paraphrase: there are many ways things could have been besides the way that they actually are. On the face of it, this sentence is an existential quantification. It says that there exist many entities of a certain description, to wit, 'ways things could have been'. I believe things could have been different in countless ways. I believe permissible paraphrases of what I believe; taking the paraphrase at its face value, I therefore believe in the existence of entities which might be called 'ways things could have been'. I prefer to call them 'possible worlds'. (Lewis 1973: 84)

Possible Worlds without 'Extreme' Modal Realism

› An alternative account is this (Stalnaker 1976; Kripke 1980: 43–46): a possible world is **a way that (actual) things might have been**, but that isn't a concrete thing (akin to 'I and all my surroundings') – it's a **property** that I and all my surroundings might have had.

  » Talk of *ways* is often talk of properties, e.g, RED is a way for things to be coloured.

› We start with the things we can actually talk about in our current language and we can make up some stories about those things in our language. If the story could have been true, then the way the story (falsely) says things are is a way things could have been, and is thus a possible world – a property that our world could have had, and which determines what would have been true if our world had turned out to have it.

› This is a kind of **realism**: but we commit to abstract uninstantiated properties, not to a pluriverse of real talking donkeys, etc.

> *The way things are* is a property or a state of the world, not the world itself. The statement that the world is the way it is is true in a sense, but not when read as an identity statement…. One could accept … that there really are many ways that things could have been … while denying that there exists anything else that is like the actual world. (Stalnaker 1976: 68)

Soames on Possible World States

> The fact that something in a logical model is called a 'world' doesn't mean that it is a concrete entity, like our universe, existing in a 'pluriverse of alternate realities'. It is enough that it be something relative to which sentences and other expressions are evaluated – a maximally complete and informative property that represents the universe as being a certain way – i.e., 'a way the world might be'.
>
> On this construal, what have been called 'worlds' are better called 'world-states'. The actual world-state is the maximal world-representing property that is instantiated; a possible world-state is one that could have been instantiated. (Soames 2010: 52)

› This 'properties view' of possible worlds is metaphysically more palatable than Lewis' framework, and will be our official account for philosophical purposes.

› Yet Lewis' theory allows a straightforward treatment of all intensions as sets of actual and possible individuals/worlds – so sometimes we'll use that kind of approach for convenience. (That way of talking can be translated into the language of functions from worlds to extensions and thus rendered compatible with the Stalnaker-Kripke view.)

Propositional Attitudes

Objection: Necessary and Impossible Propositions

Some sentences, it is widely supposed, are *necessarily true*: they could not have failed to be true, and so, in a paraphrase widely accepted among philosophers, they are true in every possible world. *Two plus three equals five*, a defender of this theory will urge, is surely true in every possible world…, and the same applies to *Three plus four equals seven*. But if *Two plus three equals five* and *Three plus four equals seven* are both true in every possible world, the theory we have been looking at is forced to say that the meaning of each of them is the set of all possible worlds. The theory predicts, in other words, that these two sentences have the same meaning.… The same kind of objection can be launched using sentences that are *necessarily false* [which] too are predicted to have the same meaning, namely the empty set, the unique set that has no members. (Elbourne 2011: 51)

Belief and the Attitudes

› The problem is worse. Since propositions are also the objects of the propositional attitudes, it also turns out that whenever someone believes one necessary proposition, they believe them all, since there is only one.

» This follows from the Relational Analysis, which was part of the evidence for referentialist propositions.

› So someone who believes that two plus three equals five, also believes that all vixens are vixens, because both express the universal proposition, the set of all worlds:

> The problem is that the possible worlds analysis seems to have the following paradoxical consequence: if a person believes that *P*, then if *P* is necessarily equivalent to *Q*, he believes that *Q*. (Stalnaker 1984: 72)

› But this seems **implausible**.

Distribution over Conjunction

› Note that many attitudes **distribute over conjunction**: so if S knows $P \wedge Q$, S knows P *and* S knows Q.

  » Likewise with *believes*, *asserts*, *says*,….

› Let us say say that Q is a **necessary consequence** of P iff every possible world in which P is also a possible world in which Q.

  » So any necessary truth is a necessary consequence of anything; any logical consequence of P, no matter how complex, is a necessary consequence of P.

› Note that if Q is any necessary consequence of P, then $P \wedge Q$ is true in exactly the same worlds as P: $[\![P]\!] = [\![P \wedge Q]\!]$. The problem now is fairly immediate:

  (8) S believes P.
  (9) S believes $P \wedge Q$. (From 8 given how Q was just defined.)
  (10) S believes P and S believes Q (From 9, distribution)
  (11) S believes Q (From 10, conjunction elimination)

› So now anyone who believes anything at all believes every necessary consequence of it, including every necessary truth (Soames 1987: 48–50).

› Can the simple view that identifies propositions and intensions respond?

Stalnaker's Metalinguistic Approach

> The expressions 'that P' and 'that Q' are schemas for sentential complements which denote propositions. The statement 'P is necessarily equivalent to Q', however, is a schema for a claim about the relation between *two expressions*. Hence here the letters P and Q stand in for expressions that denote things that express the proposition that P. Now once this is recognized, it should be clear that it is not part of the allegedly paradoxical consequence that a person must know or believe that P is equivalent to Q whenever P *is* equivalent to Q. When a person believes that P but fails to realize that the *sentence* P is equivalent to the sentence Q, he may fail to realize that one of the propositions he believes is expressed by that sentence. In this case, he will still believe that Q, but will not himself put it that way. And it may be misleading for others to put it that way in attributing the belief to him. (Stalnaker 1984: 72)

Stalnaker's Semantics for Belief

› Maybe when *S* believes that *P*, *S* stands in the belief relation to the proposition expressed by *P expresses a truth*.

> If sentence *s* expresses … proposition *P*, then the second proposition in question is the proposition that *s* expresses *P*. In cases of ignorance of necessity and equivalence… it is the second proposition that is the object of doubt and investigation. (Stalnaker 1984: 84–85)

» Even though some unproved theorem of mathematics expresses the necessary proposition, and I believe that proposition, it is false to say I believe the unproved theorem. For I do not believe **of** the **sentence** which expresses the unproved but true theorem, **that** it expresses the unproved but true theorem.

› In effect, Stalnaker swaps the necessary proposition as the object of belief with a contingent **meta-linguistic** proposition. And 'it may be a nontrivial problem to see what proposition is expressed by a given sentence' (Stalnaker 1984: 84; see also Stalnaker 1987).

Alternative: Impossible Worlds

› Elbourne (Elbourne 2011: 53–54) considers whether to treat intensions as sets of generic 'truth-supporting circumstances' (Soames 1987) – entities at which propositions can be true but which need not be possible worlds – including partial worlds, or **situations** (Barwise and Perry 1983), and **impossible worlds** (Nolan 1997).

› An impossible world is **a way things couldn't have been**; it need be neither **complete** nor **consistent**.

› If P and Q are necessarily equivalent, it is impossible that one be true and the other false. But even if it is impossible, there is an impossible world i where one is true and the other false. So the meaning of P is distinct from the meaning of Q, because one classifies i as true, the other classifies it as false.

  » Since the meaning is different, no problem with belief.

› Problem: It is impossible that there be a **male vixen**. So there is an impossible world w_i in which there is a male vixen. The sentence *All vixens are female foxes* is false at w_i, while *All vixens are vixens* is true at w_i. Different intension, different meaning. But now the theory is too powerful, since it enables differences in meaning between **synonymous** sentences, which should not differ in meaning!

Negative Polarity Items

Evidence for the Possible Worlds Account of Propositions

› There is *prima facie* evidence for the possible worlds account of propositions; and there are some proposals for responding to some obvious challenges.
› But the more persuasive evidence in favour of the proposal doesn't seem to come from these general philosophical observations.
› Instead, the proof is in the pudding: that the possible worlds account of propositions enables us to explain otherwise mysterious linguistic phenomena is some evidence for it. (Particularly if rival accounts haven't yet managed a satisfactory explanation.)
› Elbourne (Elbourne 2011: 55–65) argues that the theory of **negative-polarity items (NPIs)** provides a case where the possible worlds explanation is successful, unexpected, and not yet matched by rival theories of propositions.

Some More Detail About Set Theory

› A set is a **collection** of things, its **members**.

Some features of sets (see fig. 1)

› If A and B are sets, and all the members of A are members of B, then A is a **subset** of B, written $A \subseteq B$.
› If A is a set, and B is a set, and all the members of A are members of B, and *vice versa* – so that A is a subset of B and B is a subset of A, and hence that A and B have the same members – then $A = B$.
  » This is the principle of **extensionality**
› If A and B are sets, then there is a set which has as its members just those things in **both** A and B, called the **intersection** of A and B, written $A \cap B$.
› If A and B are sets, then there is a set which has as its members just those things in **either** A or B, called the **union** of A and B, written $A \cup B$.
› If A and B are sets, then there is a set which has as its members just those things in A which are **not** in B, called the **complement** of B with respect to A, written $A \setminus B$.
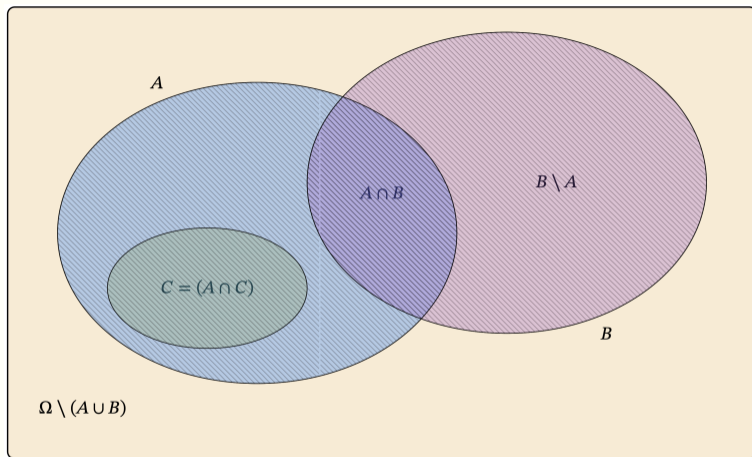
A picture is worth ≈200 words

Figure 1: Sets and their relations, showing sets A, B, and C, where $C \subseteq A$, $A \cup B$ is the diagonally hatched area, $A \cap B$ in purple, $B \setminus A$ in pink, and the complement of $A \cup B$ in cream.

Intensions and Sets

› We will continue with our working hypothesis, that the meaning of a sentence is an intension, a set of possible worlds. Other expressions don't have sets as their meanings; they, instead, denote **operations on sets**.

› Some are simple: take *and*. This is an operator that, given two sentences, forms a complex sentence. Where P, Q are sentences, we see that $[\![P \text{ and } Q]\!] = [\![P]\!] \cap [\![Q]\!]$. So $[\![\text{and}]\!] = \cap$.

Adjectival Modification (Morzycki 2016: §2.2)

› Interestingly, this set-theoretic approach also helps with adjectival modification as in *Tom is a Candadian surgeon*, where ⟦Canadian surgeon⟧ = ⟦Canadian⟧ ∩ ⟦surgeon⟧.

» These are called, unsurprisingly, **intersective adjectives**. (Remember *pet fish*?)

› But not all adjectives have this feature: consider *Olga is an experienced surgeon*, which does not entail that Olga is experienced in general – unlike *Olga is a Russian surgeon*, which does entail that she is Russian in general.

› In this case, ⟦experienced surgeon⟧ ⊆ ⟦surgeon⟧. This is a **subsective** adjective.

» All intersective adjectives are also subsective, but not *vice versa*.

› But this is not generalisable, as modifiers like *fake* or *apparent* seem to be non-subsective.

NPIs

› A negative-polarity item is, roughly, a word that is happiest in a **negative environment**. Here are some examples (Ladusaw 1980: 457):

(12) Chrysler dealers don't ever sell any cars anymore.
(13) *Chrysler dealers ever sell any cars anymore.
(14) The 6:05 hasn't arrived yet.
(15) *The 6:05 has arrived yet.
(16) No student has arrived yet.
(17) *Some student has arrived yet.

› In these examples, *ever*, *any*, and *yet* are NPIs: they are comfortable in the negative environments – in the scope of apparently negative words like *don't*, *hasn't*, and *no student* – found in the sentences (12)–(16), but awkward in the sentences (13)–(17).

Characterising NPIs

› The problem is that, despite their name, NPIs are also happy in a bunch of non-negative environments too:

(18) John forgot to return **any** of the money;

(19) Mary is too smart to have started **yet**;

(20) Every student who had **ever** read **any** Quine passed;

(21) At most three students **ever** read **any** Quine.

› So it is not obvious how to characterise NPIs: it is not the presence of an overt 'negation' operator like *not* or *no*.

› Indeed, it seems implausible that it be syntactic at all: witness these non-negative but syntactically identical cases:

(22) It's surprising that (some/any) money was taken;

(23) It's plausible that (some/*any) money was taken.

› It must be something about the **meaning**.

Preliminaries to our answer

› It must be something about the meaning of expressions like *forgot*, *too smart*, *every student*, *surprising*.

› Let's follow Elbourne, and concentrate on two sorts of expressions:

  » **Determiner phrases**, like *Every student*, *No god*, etc.
  » **Sentential operators** like *not* ('it is not the case that …'), *never* ('it is never the case that …'), *It is surprising that*, etc.

› When a sentence involving an NPI is grammatical, it must be because of the presence of some expression that permits the presence of an NPI. That expression will be said to **license** the NPI.

› So our question is: which determiner phrases and sentential operators license NPIs?

Downward Entailment

› Ladusaw's answer is: those expressions which are **downward entailing**.

› Here's Elbourne: an expression

> is downward entailing if and only if, for all phrases A and B, if the meaning of B is a subset of the meaning of A, then the sentence composed of O and A entails the sentence composed of O and B. (Elbourne 2011: 58)

› More formally, an expression O is downward entailing iff, for expressions A and B with sets as their intension, if $[\![B]\!] \subseteq [\![A]\!]$, then $[\![O(A)]\!] \subseteq [\![O(B)]\!]$ (Ladusaw 1980: 467).

Example: *Not*

› *Not* is downward entailing. It is a sentential operator, and what it contributes to sentences in which it appears is the set theoretic **complement** operator ('\\').
  » That is, when S is some sentence expressing a proposition – set of worlds – $[\![S]\!]$, *not-S* expresses $\Omega \setminus [\![S]\!]$, that is, the set of all worlds not in $[\![S]\!]$.

› So what if S entails T, so that the meaning of S is a subset of the meaning of T: $[\![S]\!] \subseteq [\![T]\!]$?
  » Intuitively, a logically stronger proposition corresponds to a smaller set of worlds – it rules out more.

› Then the meaning of *not-T* will be a subset of the meaning of *not-S*, and *not-T* **entails** *not-S*, as in the definition. Again intuitively: if S entails T, then if T is false, S must be too, i.e., if *not-T* is true, so is *not-S*.
  » Compare Elbourne on *never* (2011: 60–61).

Example: *No student*

› *No* is a **quantifier**. The standard treatment of quantifiers in formal semantics is a little different from treatments you might have seen in logic.

› Consider a sentence like *No student fails*. It has the form *No X Ys*, where X is the predicate part of the determiner phrase, and the predicate Y is its subject.

› That is: *No X* is a **predicate of a predicate** Y: Y satisfies it at those worlds where Y doesn't overlap with X.

› The semantic clause is thus something like this:

$$[\![\text{No } X\, Y]\!] = \Big\{ w \,:\, \big([\![X]\!] \cap \mathsf{Dom}(w)\big) \cap \big([\![Y]\!] \cap \mathsf{Dom}(w)\big) = \varnothing \Big\}.$$

  » A world w is in the intension of this proposition just in case the extensions of X and Y at that world have an empty intersection, or are **disjoint**. (Here '$\mathsf{Dom}(w)$' is the set of things that exist at w, its **domain**.)

  » Note here we are using the idea from lecture 2 that the intension of a predicate is a set of actual and possible entities; for this to be a well-behaved set, worlds must not overlap in their contents – they are Lewisian concrete universes. A more ecumenical approach takes a predicate intension to be a function from worlds to extensions.

› So the intension of *No student fails* is the set of worlds w where the (relevant) students in w don't overlap with those who fail in w.

Downward Entailingness of *no student*

› Given these semantics, *no student* is downward entailing.
› *no student* denotes the property that a predicate A has at w just in case the students and the As don't overlap at w.
› Now suppose $[\![B]\!] \subseteq [\![A]\!]$ – if so, the extension of B in any world is a subset of the extension of A.
› So if A has the property expressed by *no student*, B will have it too: if the Bs are wholly among the As, then they can't overlap the students without the As doing so as well.
› So the set of worlds where no student As must be included in the set of worlds where no student Bs, and hence *no student As* entails *no student Bs*. So that phrase meets the condition for a downward-entailing environment: i.e., if $[\![B]\!] \subseteq [\![A]\!]$, then $[\![\text{No student}(A)]\!] \subseteq [\![\text{No student}(B)]\!]$.

  » Since $[\![\textit{fails spectacularly}]\!] \subseteq [\![\textit{fails}]\!]$, *No student fails* entails *No student fails spectacularly*.

Example: *At most three students*

› *At most three* (as in the NPI-licensing 21) is also a **quantifier**, parallel in form to *No*.
› The semantic clause is something like this:

$$\llbracket \text{At most three } X\, Y \rrbracket = \left\{ w : \left| \big(\llbracket X \rrbracket \cap \mathsf{Dom}(w)\big) \cap \big(\llbracket Y \rrbracket \cap \mathsf{Dom}(w)\big) \right| \leqslant 3 \right\},$$

where $|A|$ denotes the **cardinality** of the set A, the number of members it has.

  » A world w is in the intension of this proposition just in case the overlap of X and Y at that world has no more than three membrs.
  » So the intension of *At most three students skateboard* is the set of worlds w where the (relevant) students in w who are within those who skateboard in w number between 0 and 3.

› Given these semantics, *at most three students* is downward entailing.
› Again suppose $\llbracket B \rrbracket \subseteq \llbracket A \rrbracket$; hence if A has the property expressed by *at most three students*, B will have it too: if the Bs are wholly among the As, there can't be more than 3 of them among the students without more than 3 As doing so as well.
› Hence *at most three students A* entails *at most three students B*.

  » Since $\llbracket \textit{skateboard well} \rrbracket \subseteq \llbracket \textit{skateboard} \rrbracket$, *At most three students skateboard* entails *At most three students skateboard well*.

In Favour of Possible Worlds Semantics

› We can adapt this story to some of our other unexpected cases, such as the NPI-licensing *surprise* (22) vs the non-licensing *plausible* (23).

  » Intuitively, *p* is surprising iff it is improbable; if *q* entails *p*, *q* must also be improbable (theorem of probability theory). So *surprising that* is a downward-entailing environment. Yet plausible things can be entailed by implausible things, so *plausible that* is not downward entailing.

› The results speak in favour of the possible worlds theory of propositions: that theory is **able to explain**, relatively simply, the facts about NPI licensing.

› To give a **uniform and systematic** theory, we needed entailment to be a subset-like relation (which of course it is on the possible worlds theory of propositions).

› So to that extent it speaks in favour of that conception as an adequate **model** of natural language meaning: it is the only sort of theory on which a uniform theory of the very broad range of NPI-licensors can be given.

References

References

Barwise, Jon and John Perry (1983/1999) *Situations and Attitudes*. CSLI Publications.

Elbourne, Paul (2011) *Meaning: A Slim Guide to Semantics*. Oxford University Press.

Frege, Gottlob (1892/1993) 'On Sense and Reference', in Adrian Moore, ed., *Meaning and Reference*: 23–42. Oxford University Press.

Kripke, Saul (1980) *Naming and Necessity*. Harvard University Press.

Ladusaw, William A (1980/2002) 'On the Notion *Affective* in the Analysis of Negative-Polarity Items', in Paul Portner and Barbara H Partee, eds., *Formal Semantics: The Essential Readings*: 457–70. Blackwell.

Lewis, David (1973) *Counterfactuals*. Blackwell.

Lewis, David (1986) *On the Plurality of Worlds*. Blackwell.

References (cont.)

McGrath, Matthew and Devin Frank (2024) 'Propositions', in Edward N Zalta and Uri Nodelman, eds., *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/fall2024/entries/propositions/.

Morzycki, Marcin (2016) *Modification*, Key Topics in Semantics and Pragmatics. Cambridge University Press.

Nolan, Daniel (1997) 'Impossible Worlds: A Modest Approach', *Notre Dame Journal of Formal Logic* **38**: 535–72. doi:10.1305/ndjfl/1039540769.

Schiffer, Stephen (2003) *The Things We Mean*. Oxford University Press.

Soames, Scott (1987) 'Direct Reference, Propositional Attitudes, and Semantic Content', *Philosophical Topics* **15**: 47–87. doi:10.5840/philtopics198715112.

Soames, Scott (2010) *Philosophy of Language*. Princeton University Press.

Speaks, Jeff (2014) 'What's Wrong with Semantic Theories Which Make No Use of Propositions?', in *New Thinking about Propositions*: 9–24. Oxford University Press.

References (cont.)

Stalnaker, Robert C (1976) 'Possible Worlds', *Noûs* **10**: 65–75. doi:10.2307/2214477.

Stalnaker, Robert C (1984) *Inquiry*, A Bradford Book. MIT Press.

Stalnaker, Robert C (1987) 'Semantics for Belief', *Philosophical Topics* **15**: 177–90. doi:10.5840/philtopics198715116.