

# Omniscience and Freedom

Antony Eagle

University of Adelaide  
<[antony.eagle@adelaide.edu.au](mailto:antony.eagle@adelaide.edu.au)>

God, Faith and Infinity » Lecture 7

# Contents

- Omniscience
- Freedom and Foreknowledge
- Unknowable Truths
- Size Problems
- Indexical Knowledge

# Omniscience

# Defining Omniscience

- › To be **omniscient** is to know **everything**.
- › Well, not really. An omniscient being need not know:
  1. Everyone or everything (so we exclude so-called **knowledge by acquaintance**); or
  2. How to do any task (so we exclude so-called **knowledge how**); or
  3. Anything which is actually false (since **knowledge that is factive**).
- › So we arrive at this definition:

(OS) Necessarily, X is omniscient iff for every true proposition  $p$ , X knows that  $p$ .
- › Traditionally, God is conceived as omniscient, because all truths are **directly available** to his view.

# Boethius on God, Eternity, Atemporality

All who live by reason agree that God is eternal, and we must therefore think about what eternity means. This will clarify what the divine nature is and also what divine knowledge must be. Eternity is the whole, simultaneous, perfect possession of limitless life... it has knowledge of the whole of life, can see the future, and has lost nothing of the past. It is in an eternal present and has an understanding of the entire flow of time. ...

It is one thing to proceed through infinite time, as Plato posits, but quite another to embrace the whole of time in one simultaneous present. This is obviously a property of the mind of God. ...

The endless and infinite changing of things in time is an attempt to imitate eternity, but it cannot equal its immobility and it fails to achieve the eternal present, producing only an infinite number of future and past moments. ... if we use proper terms, then ... we should say that God is eternal but the world is perpetual.

(Boethius 524 CE: 168-70).

# Divine Omniscience

- › For Boethius, God's **eternity** is like **atemporality**. He is not totally outside of time – not like the numbers or other abstract objects, but is outside of the flux of time, unchanging.
- › Because all facts are **permanently present** to his mind, regardless of when they occur, he **knows** all of them.
- › They are permanently present because everything in God's experience is in effect 'simultaneous', having no temporal structure, from the divine perspective.
  - › God knows of course the temporal relations between events – it's just that his **experience of** those events lacks any corresponding temporal structure.
  - › Likewise his mental states: 'if God is timeless, God's existence and believings have no temporal location' (Leftow 1991: 311).
- › A **perpetual** being would instead have infinitely many successive experiences.
  - › Presumably analogous claims go for space: God is not everywhere, but 'embraces the whole of space in one coincident here'. God knows everything about other times and places because there is no spatiotemporal structure to his experience.

# Is God a Knower?

- › Can an atemporal being be a knower? Many would say **no**. E.g., Goldman offers an early influential **causal analysis** of knowledge:
  - S knows that *p* if and only if the fact *p* is causally connected in an 'appropriate' way with S's believing *p*. (Goldman 1967: 369)
- › If God is not 'in time', but rather **eternal** in the way Boethius describes, then God cannot know anything – if this analysis is correct.
- › Even if we reject the causal analysis, many place causal (or at least temporal) **constraints** on particular kinds of knowledge. E.g.
  - it is *simply impossible* for an agent to have knowledge of the outcome of an objectively chancy event if there is no flow of information from the event to the agent. (Oppy and Scott 2010: 254)

# Evidentialist Conceptions of Knowing

- › On the other hand, many have argued that knowing (or even ‘a flow of information’) doesn’t require causation – it only requires beliefs that are suitably matched with the **evidence**.
- › **Strong and undefeated** evidence for a true belief suffices for that belief to be knowledge, regardless of causal connection.
  - › E.g., suppose you form a true belief that it will rain today from the weather forecast. But the forecast isn’t an effect of today’s rain – it is causally downstream of **yesterday’s** weather.
  - › So you have **some evidence** of today’s rain (the forecast), which suffices for knowledge of today’s rain – yet the fact that it is raining today is not causally connected to that knowledge.
- › If God always has good evidence, then God can know by **inference** even if God is outside of time.

# Freedom and Foreknowledge

# Another View of Permanent Presence

- › The alternative view is that God is, in Boethius' words, **perpetual**: eternal but in time, and yet still all those facts are permanently present to his mind. Pike attributes this view to Calvin:
  - with respect to any given natural event, not only is that event 'present' to God in... it has ever been and has perpetually remained 'present' to Him....
  - Whatever one thinks of the idea that God 'sees' things as if 'actually placed before him,' Calvin would appear to be committed to the idea that God has always known what was going to happen in the natural world. Choose an event ( $E$ ) and a time ( $T_2$ ) at which  $E$  occurred. For any time ( $T_1$ ) prior to  $T_2$ ..., God knew at  $T_1$  that  $E$  would occur at  $T_2$ . ... Calvin says, 'when God created man, He foresaw what would happen concerning him.' (Pike 1965: 30)
- › So God is at every point in time, and his knowledge at each point in time reflects what is happening at every other point. He **always knows the same things**, but there is nevertheless **temporal structure** to God's existence.
  - › It is worth thinking of Jesus here, who is said to be both a man and (given the Trinity) God; Jesus was certainly not outside the flow of time, but he did know, e.g., that Judas would betray him ([Matthew 12:22-3](#)).

# Free Action

- › This second, Calvinist view of God's perpetuity generates some problem cases.
- › These are cases involving **libertarian free will**.
- › For if X exercises libertarian free will at  $t$ , then it is **not determined** by any past history or laws how their exercise of free will turns out.
- › And many have thought that God could not know the outcomes of such free choices before they happen – **for if the outcomes were known prior to the choices being made, they would not have been free**.
- › So God cannot be omniscient until the final exercise of free choice.
  - › Particularly noteworthy if we have opted for a free will theodicy in response to the problem of evil.
- › **None of this applies** if God is eternal in Boethius' sense: for then God's experience and mental states have no temporal structure, and hence no part of God's knowing stands in any temporal relations to any free actions. If 'God's existence and believings have no temporal location, they are not before [anyone]'s action' (Leftow 1991: 312).

## Is this a Genuine Issue?

- › Consider a more mundane case: Beth tells Alfred, her colleague, that she is going away on holiday, and won't be in the office next week.
- › Alfred, knowing Beth and with no reason to suspect her of lying, forms the belief that Beth will not be in the office the following week.
- › In fact, Beth is sincere and cooperative; she will in fact be away.
- › Then Alfred's belief looks like a clear case of (testimonial) knowledge.
  - › It's true, it's believed, it is reasonable in light of the evidence, etc.
- › But now: Alfred knows Beth will be away the following week. It follows, from the factivity of knowledge, that Beth **will** be away the following week.
- › But this seems to pose no threat to Beth's freedom. She can freely choose to give up her holidays at any point; the fact that she **will not do so** doesn't entail that she **cannot do so**.
  - › To suppose that anything that will happen, must happen, is the characteristic error of **fatalism**.
- › Is the threat of divine foreknowledge simply the mistake of fatalism again?

## Pike's argument (1965: 33–34)

- (P<sub>1</sub>) God is omniscient at  $t_1$ .
- (P<sub>2</sub>) If Jones does X at  $t_2$  (later than  $t_1$ ), then God knows at  $t_1$  that Jones does X at  $t_2$ . (From P<sub>1</sub>)
- (P<sub>3</sub>) If God knows at  $t_1$  that Jones does X at  $t_2$  and Jones has the power to refrain from doing X at  $t_2$ , then 'it was within Jones's power at  $t_2$  to do something that would have brought it about that' either:
  - 1. 'God held a false belief at  $t_1$ '; or
  - 2. 'God did not hold the belief He held at  $t_1$ '; or
  - 3. 'God ... did not exist at  $t_1$ '.
- (P<sub>4</sub>) No one has the power to do something that would bring about any of the things mentioned under (P<sub>3</sub>).
- (P<sub>5</sub>) So if Jones does X at  $t_2$ , Jones cannot refrain from doing X at  $t_2$ . (P<sub>2</sub>–P<sub>4</sub>)
- (P<sub>6</sub>) So Jones' doing X at  $t_2$  would not be free or voluntary. (from P<sub>5</sub>)

# Defending (P3)

- › Informally, (P3) suggests that refraining from doing something God knows you will do would **falsify** God's past belief.
- › And that, Pike thinks, is either to do something contradictory to factivity of knowledge; or to change the past, either by changing God's mental state or God's existence.
- › Suppose I know now that you will come to class tomorrow.
  - › Perhaps I can **predict with certainty** your attendance.
- › Then can you do otherwise? **Apparently not.**
  - › If I **know** it now, it is now true that you will come to class.
  - › So the only way for you to make it false that you attend tomorrow is to do something tomorrow that ensures I **don't** know it today – by changing me or my evidence.
  - › But it is impossible to **change** the past; so you don't have the power to do so.

## (P4) expanded

3. It is not within one's power at a given time to do something having a description that is logically contradictory.
  4. It is not within one's power at a given time to do something that would bring it about that someone who held a certain belief at a time prior to the time in question did not hold that belief at the time prior to the time in question.
  5. It is not within one's power at a given time to do something that would bring it about that a person who existed at an earlier time did not exist at that earlier time. (Pike 1965: 33-34)
- › This looks very plausible – how could a free agent have the power to change the past or to make a contradiction true?
  - › Nevertheless, there are conceptions of free action and knowledge that cast considerable doubt upon (P4) – while accepting that the past cannot be changed.

# Changing the Past – Time Travel Scenarios

But the events of a past moment are not subdivisible into temporal parts and therefore cannot change. Either the events of 1921 timelessly do include Tim's killing of Grandfather, or else they timelessly don't. We may be tempted to speak of the "original" 1921 that lies in Tim's personal past, many years before his birth, in which Grandfather lived; and of the "new" 1921 in which Tim now finds himself waiting in ambush to kill Grandfather. But if we do speak so, we merely confer two names on one thing. The events of 1921 are doubly located in Tim's (extended) personal time, like the trestle on the railway, but the "original" 1921 and the "new" 1921 are one and the same. If Tim did not kill Grandfather in the "original" 1921, then if he does kill Grandfather in the "new" 1921, he must both kill and not kill Grandfather in 1921 - in the one and only 1921, which is both the "new" and the "original" 1921. It is logically impossible that Tim should change the past by killing Grandfather in 1921. So Tim cannot kill Grandfather. (Lewis 1976: 150)

# Knowledge from Future Testimony

- › Maybe I know what you will do tomorrow because your future autobiography fell through a **wormhole** and I read it.
- › Then I am not deducing tomorrow's events from today's causes of those events – I am deducing tomorrow's events from the future **effects** of those events, which I just happen to have access to.
- › Here there is **causal dependence** of the past on the future – again, not changing the past, but the future plays a role in causing the past to be as it was.
  - › No **loops** here, because my knowing what you will do doesn't play any role in causing you to do it. So you are able to cause it however you like – including by an exercise of libertarian free will, if you have that capacity, or by any other exercise of free choice.
  - › If you had exercised your free will differently, my mental state would have been caused differently, and hence you would have done something that ensures the past always was different, without changing the past (without, that is, the past being one way yesterday and altered to be a different way today.)

# God's Knowledge of the Uncertain Future

- › So even if you have libertarian free will, someone can know the outcome of your action if their knowledge **depends** (in the right way) on that outcome.
- › For us, the 'right way' is still **causal** – and since we don't typically have access to the newspapers and biographies of the future, libertarian free choices and objectively chancy outcomes are **unknown** to us.
- › But why think God is restricted to causal access to the future?
  - if we are prepared to suppose that there can be non-physical transmission of information – and which theists would wish to deny this? – then ... we can suppose that, at any time, we can truly say that God knows the outcomes of objectively chancy events that lie in our future. ... what God believes now *depends upon* the upshot of the free decision that Jones makes next Saturday. (Oppy and Scott 2010: 254-55)

# Laws and Knowledge

- › Even though the past isn't under your control, **the laws might well be**.
- › If whether or not  $X$  is under your control, then it is under your control whether the laws say 'if the past is such-and-such,  $X$ ' or instead 'if the past is such-and-such, not- $X$ '.
  - › If you have the power to refrain from attending class tomorrow, then you have the power to **influence** the laws stating how your coming to class depends on the past history. You thus could have the power to do something that, if you did it, would ensure an actual law wouldn't (counterfactually) have been a law (Lewis 1981).
- › So if you can refrain from coming to class, you influence the laws, because **claims about laws are not claims wholly about the past**, but about the whole past and future history of the world.
- › If God **deduces** your future behaviour from the past and laws jointly, then your action that fixes what the laws are influences what God knows.
  - › If the laws are part of God's evidence, you can change God's evidence without changing the past – since **part of God's evidence is about the future**, which is under your control.

## Revisiting (P4)

- › So we now see that, in a number of different circumstances, (P4) is false.
- › In particular, it is possible for us at a time to ‘do something that would bring it about that someone who held a certain belief at a time prior to the time in question did not hold that belief at the time prior to the time in question.’
- › If their having the belief at that time depends upon what we do – either (i) causally, (ii) via our influence over the laws, or (iii) via whatever non-causal dependence is involved in divine knowledge – then if we can do something different now, we can ensure that they would always have had a different belief than the one they actually have.
- › **This isn’t changing the past – it is actualizing another possibility in which the past would have been different *all along*.**
- › This **Ockhamist** solution to the problem of divine foreknowledge is historically traced back to William of Ockham (Zagzebski 2017: §2.3).

## Denying the inference from (P5) to (P6)

Black ... wishes to see White dead but is unwilling to do the deed himself. Knowing that Mary Jones also despises White ... Black inserts a mechanism into Jones's brain that enables Black to monitor and to control Jones's neurological activity. If the activity in Jones's brain suggests that she is on the verge of deciding not to kill White when the opportunity arises, Black's mechanism will intervene and cause Jones to decide to commit the murder. On the other hand, if Jones decides to murder White on her own, the mechanism will not intervene. ... Now suppose that when the occasion arises, Jones decides to kill White without any "help" from Black's mechanism. ... Jones is morally responsible for her act. Nonetheless, it appears that she is unable to do otherwise since if she had attempted to do so, she would have been thwarted by Black's device. (Zagzebski 2017: §2.5)

- › Jones cannot refrain; still, Jones was responsible for the act. (Frankfurt 1969).
- › Since **responsibility requires freedom**, Jones was free (i.e., P5 true, P6 false).
  - » However: the truth of (P5) is already a problem for a free will theodicy. Many theists will wish to resist the argument for (P5), regardless of (P6).

# Unknowable Truths

# Self-Reference and Unknowability

- › Another source of unknowable truths might be **paradoxical sentences**. Grim gives an example based on the Liar – Routley (2010: n. 23) discusses a related case:

(1) X does not believe that (1) is true.

For X we substitute any name or referring expression. For any such substitution we can ask whether (1) is true or false. If (1) is true, X does not believe it, and thus X cannot be said to know all truths. If X is false, it is false that X does not believe it. It must then be true that X believes it. X therefore believes a falsehood.

The trap is set. In light of (1), there is no X that can qualify as omniscient. For any candidate put forward, there will be a form of (1) that must be either true or false. But if true, the candidate cannot qualify as omniscient. If false, the candidate cannot qualify as omniscient. There can be no omniscient being, and thus can be no God. (Grim 2013: 169-70)

## Another Example: the Knower

- › The foregoing argument is not quite successful against omniscience **defined as in (OS)**.
- › For a being could manage to believe all truths, and thus be omniscient, while believing some falsehoods too.
  - › A perfect god is supposed to be omniscient and **infallible**, so it is a problem for perfect being theism still.
- › If we want to show, using self-reference, that omniscience is impossible, we'll need a slightly different case.
- › Consider this **self-referential** sentence:
  - (K) No one knows that this sentence is true. (Tymoczko 1984: 437)
- › This can be formalised as the **paradox of the knower** (Kaplan and Montague 1960).

# Logic and Formalisation

- › **Formalising** these self-referential sentences is important, to be sure that the paradox isn't arising because we are being imprecise or sloppy in our thinking.
- › Here is a quick-and-dirty guide to the **logical notation** we'll need:
  - ›  $\ulcorner, \urcorner$  Logical **quotation marks**. For any sentence  $\phi$ ,  $\ulcorner \phi \urcorner$  is a name of  $\phi$ : just as 'Antony' names me, and "'Antony'" is a name of my name.
  - ›  $\neg, \wedge$  Logical symbols: For any sentence  $\phi$ ,  $\ulcorner \neg \phi \urcorner$  is its **negation**, the sentence which is true if and only if  $\phi$  is false; and for any  $\phi$  and  $\psi$ , their **conjunction**  $\ulcorner \phi \wedge \psi \urcorner$  is true iff both conjuncts are true.
  - ›  $\vdash, \dashv\vdash$  The **derivability** symbols:  $\Gamma \vdash \phi$  means that you can prove  $\phi$  given the sentences in  $\Gamma$ .  $\phi \dashv\vdash \psi$  means that you can derive  $\phi$  from  $\psi$  and *vice versa*.
- › We'll also display our proofs in a **natural deduction** format; this is a proof system that helps us keep track of what assumptions are involved at various points in an argument.

# The Knowledge Predicate

- › We need three ingredients to turn this into a genuine paradox.
- › The first ingredient is a **knowledge predicate**.
- › We will assume that there is in our language a predicate of sentences  $\mathcal{K}$ , such that ' $\mathcal{K}\ulcorner\phi\urcorner$ ' is intended to mean 'It is known (by someone) that  $\phi$  is true'.
- › Since it is a knowledge predicate, it is **factive**: if we can show that  $\mathcal{K}\ulcorner\phi\urcorner$  given some assumptions, we can also show  $\phi$  given those same assumptions:

**Factivity** if  $\Gamma \vdash \mathcal{K}\ulcorner\phi\urcorner$ , then  $\Gamma \vdash \phi$ .

1		$\mathcal{K}\ulcorner\phi\urcorner$	
		$\vdots$	
n		$\phi$	Factivity, 1

Figure 1: The proof rule of Factivity

# The Ingredients of Self-Reference

- › Second ingredient is a **device of self-reference**. We will assume that
  1. for every sentence of our language  $\phi$ , our language also contains a **name** of that sentence  $\ulcorner \phi \urcorner$ ; and that
  2. our language contains sentences that ‘talk about themselves’.
- › Our language contains names for sentences. Where such a name  $\ulcorner \phi \urcorner$  occurs in some sentence ‘...  $\ulcorner \phi \urcorner$  ...’, and the name is previously unattached, self-reference permits us to use the name to **label** the sentence in which it occurs.
- › In the case of the knower, here is the precise sentence we want:  **$\kappa$  says of itself that it is unknown:**

$$(\kappa) \quad \neg \mathcal{K} \ulcorner \kappa \urcorner$$

- › The key fact here is that  $\kappa$  ‘just is’  $\neg \mathcal{K} \ulcorner \kappa \urcorner$ , and hence those two should be **substitutable** for one another (i.e., they are logically inter-derivable):

**Substitution**  $\kappa \dashv\vdash \neg \mathcal{K} \ulcorner \kappa \urcorner$ .

# Omniscience formalised

- › Finally, we need a **claim expressing omniscience**.
- › Suppose **O** expresses the claim that *there is an omniscient being*.
- › Then, **in line with (OS)**, the assumption that  $\phi$  is true together with **O** should justify the claim that  $\phi$  is known.
- › So we have this proof rule

**Omn** if  $\Gamma, \mathbf{O} \vdash \phi$  then  $\Gamma, \mathbf{O} \vdash \mathcal{K}\ulcorner\phi\urcorner$ .

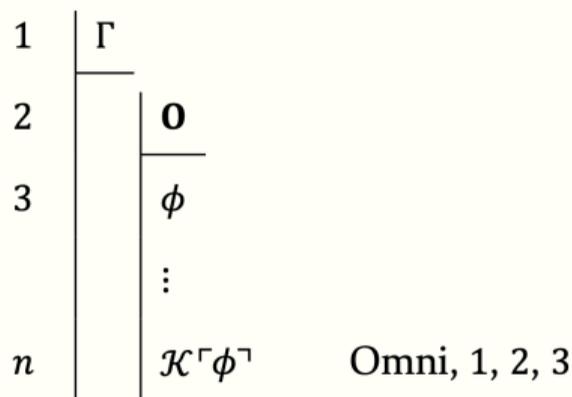


Figure 2: The proof rule of Omniscience

# A proof

- › We can use these rules to construct a formal derivation that there is no omniscient being; lines 5 and 8 use the logic of negation in our natural deduction proof system:

1			<b>O</b>	
2			$\neg \mathcal{K} \ulcorner \kappa \urcorner$	
3			$\kappa$	Substitution, 2
4			$\mathcal{K} \ulcorner \kappa \urcorner$	Omni, 3
5			$\mathcal{K} \ulcorner \kappa \urcorner$	$\neg$ E, 2-3, 2-4
6			$\kappa$	Factivity, 5
7			$\neg \mathcal{K} \ulcorner \kappa \urcorner$	Substitution, 6
8		$\neg$ <b>O</b>		$\neg$ I, 1-5, 1-7

Figure 3: A proof that **O** is false

# A Residual Problem

- One issue is that we can derive  $\kappa$  just using our rules, from no assumptions:

1		$\mathcal{K}\ulcorner\kappa\urcorner$	$\neg\text{E}, 2-3, 2-4$
2		$\kappa$	Factivity, 1
3		$\neg\mathcal{K}\ulcorner\kappa\urcorner$	Substitution, 2
4		$\neg\mathcal{K}\ulcorner\kappa\urcorner$	$\neg\text{I}, 1-1, 1-3$
5		$\kappa$	Substitution, 4

Figure 4: A proof of  $\kappa$

- This means that  $\kappa$  is an **unknown tautology**.
- To avoid further paradox, we have to reject this otherwise attractive rule:  
**Know-Intro** If  $\vdash \phi$  then  $\vdash \mathcal{K}\ulcorner\phi\urcorner$ .
- But: isn't a proof an excellent way of coming to know something?

# Unknowable Truths Without Self-Reference

- › An argument for unknowable truths that didn't involve self-reference would look a lot more plausible.
- › A famous and much discussed argument: Fitch's **knowability** argument:  
If there is some true proposition which nobody knows (or has known or will know) to be true, then there is a true proposition which nobody can know to be true. (Fitch 1963: 139)
- › The argument requires a knowledge **operator**  $\mathbf{K}\phi$  interpreted as 'it is known that  $\phi$ ', and which is **factive** and **distributes over conjunction** (i.e., if  $\mathbf{K}(\phi \wedge \psi)$  then  $(\mathbf{K}\phi \wedge \mathbf{K}\psi)$ ).
  - › A knowledge **operator** makes complex sentences that **include** simpler sentences; a knowledge **predicate** makes sentences that are **about** other sentences. ' $\mathbf{K}\phi$ ' involves no names and is not about sentences, so it is not possible to construct self-referential sentence using just this operator.
- › It also requires a possibility operator  $\diamond\phi$ , meaning 'it is possible that  $\phi$ '.

# Fitch's argument

1. Assume  $p$  is an unknown truth, so that this conjunction is true:  $(p \wedge \neg \mathbf{K}p)$ .
2. Assume that the unknowability of the unknown truth is knowable:  $\diamond \mathbf{K}(p \wedge \neg \mathbf{K}p)$ .
3. In some world:  $\mathbf{K}(p \wedge \neg \mathbf{K}p)$ . (possibly means 'in some possible world', 2)
4. In some world:  $(\mathbf{K}p \wedge \mathbf{K}\neg \mathbf{K}p)$ . (3, distribution)
5. In some world:  $(\mathbf{K}p \wedge \neg \mathbf{K}p)$ . (4, factivity of right conjunct)
6.  $\diamond(\mathbf{K}p \wedge \neg \mathbf{K}p)$  (possible world to possibly, 5)
7.  $\neg \diamond \mathbf{K}(p \wedge \neg \mathbf{K}p)$  (*reductio*, 2-6)
8. If there is an unknown truth  $p$ , there is an unknowable truth – namely, that  $p$  is an unknown truth. (conditional introduction, 1-7)

# Is there an unknown truth?

- › For this to pose a problem for God's omniscience, we need to argue that there is an unknown truth.
- › Routley (2010: 116–17) tries to argue for this; but his argument is one that will disappoint.
  1. He first argues that even God will not know some self-referential sentences, similar to those we've already discussed;
  2. He then argues that we ought to 'restrict... the class of creatures that can render something known to some (past or present) time existent creatures, roughly that is to creatures that have at some time lived (in this part of this world) or do now (so) live' (Routley 2010: 117). Why? The argument gets a little fuzzy at this point....
- › Perhaps we can try another way to showing that some proposition is unknown.
- › The next two sections aim to give examples of unknown truths; if these succeed, then Fitch's argument will establish that there is no omniscient being.

# Size Problems

# Sets and Collections

- › An omniscient being believes **every true proposition** (because knowledge entails belief).
- › That is: there is some **collection** of propositions that God believes – call it ‘Prop’ –, and they are all true.
- › If we suppose that the mathematics of collections is given by standard **set theory**, then we can prove some interesting results about this set of propositions Prop.
- › Some definitions:
  - ›› A set  $A$  is a **subset** of  $B$  iff each member of  $A$  is also a member of  $B$ . The empty set  $\emptyset$ , the set with no members, is a subset of every set.
  - ›› The **powerset** of  $A$ , written  $\wp A$ , is the set containing all the subsets of  $A$ . So if we have the set  $\{0, 1\}$ , the powerset is  $\{\{0\}, \{1\}, \{0, 1\}, \emptyset\}$ .
- › It is an **axiom** of standard set theory that every set has a powerset.

# Cantor's Theorem

**Cantor** The powerset  $\wp A$  of a set  $A$  is always bigger than  $A$ .

- › The proof is as follows.
  - › Suppose, for **reductio**, that  $A$  and  $\wp A$  are the same size. Then there is a **one-to-one correspondence**  $f$  between the members of  $A$  and the members of  $\wp A$ , i.e., between the members of  $A$  and the subsets of  $A$ .
  - › Now define a subset of  $A$  as follows: for each member  $x$  of  $A$ ,  $D$  contains  $x$  iff  $f(x)$  – which is a subset of  $A$  – does not contain  $x$ .
  - › If there is some  $a \in A$  such that  $f(a) = D$ , then  $a \in D$  iff  $a \notin D$  – contradiction! So this **diagonal** set  $D$  does not correspond to any member of  $A$ .
  - › But  $D$  is a subset of  $A$ , since it derives from sorting the members of  $A$  into two classes, and hence is in  $\wp A$ .
  - › So  $f$  is not a one-one correspondence after all.
  - › But there is a function mapping  $A$  **into**  $\wp A$ , namely  $g : x \in A \mapsto \{x\} \in \wp A$ .
  - › So  $\wp A$  must be bigger than  $A$ . (If there is a one-to-one correspondence between  $X$  and a proper subset of  $Y$ , but not between  $X$  and  $Y$ , then  $Y$  is bigger than  $X$ .)

# The Set of All Truths?

- › Consider now  $\wp\text{Prop}$ , the set containing all sets of truths God believes. Assume for *reductio* that God is omniscient.
- › For each  $S \in \wp\text{Prop}$ , there are truths about that subset of Prop. For example, either it is a truth that  $2 + 2 = 4 \in S$ , or it is a truth that  $2 + 2 = 4 \notin S$ .
- › If God believes all the truths, then God believes all the truths about the members of  $\wp\text{Prop}$ , and so the number of truths in Prop must be at least as many as the subsets of Prop, to accommodate the fact that God believes at least one truth about each subset.
  - ›› That is,  $\wp\text{Prop}$  is not bigger than Prop.
- › But by Cantor's theorem, it is.
- › Hence Prop cannot be a set of **all truths** – even if it is the set of all those truths that God believes.
- › The set of God's beliefs, Prop, cannot contain every truth about its own subsets, and so **God doesn't know those truths**.

# Against God from Unknown Truth

- › The foregoing argument is due to Grim (Grim 1988); a possible **response** is to argue that knowing all the truths doesn't require there being a collection of all the truths one believes.
- › But we could conclude instead **that there are unknown truths, even to God.**
- › If there are unknown truths, Fitch's argument allows us to conclude that there are unknowable truths.
- › And since God is **essentially** omniscient, if there are unknowable truths, we have another argument that God doesn't exist, to wit:
  1. By nature, God can know of each truth that it is true.
  2. But nothing can have that nature if there are truths that cannot be known to be true.
  3. So God can't exist.

# Indexical Knowledge

# The Essential Indexical

I once followed a trail of sugar on a supermarket floor... seeking the shopper with the torn sack to tell him he was making a mess. With each trip around the counter, the trail became thicker. But I seemed unable to catch up. Finally it dawned on me. I was the shopper I was trying to catch.

I believed at the outset that the shopper with a torn sack was making a mess. And I was right. But I didn't believe that *I* was making a mess. That seems to be something I came to believe. And when I came to believe that, I stopped following the trail around the counter.... My change in beliefs seems to explain my change in behavior.... At first characterizing the change seems easy. My beliefs changed, didn't they, in that I came to have a new one, namely, *that I am making a mess*? But things are not so simple.

The reason they are not is the importance of the word 'I' in my expression of what I came to believe. When we replace it with other designations of me, we no longer have an explanation of my behavior.... It seems to be an *essential* indexical. (Perry 1979: 3)

# Omniscience and the Essential Indexical

In order to qualify as omniscient or all-knowing, a being must know at least all that is known. Such a being must, then, know what I know in knowing (1):

(1) I am making a mess.

But what I know in such a case, it appears, is known by no omniscient being. The indexical 'I', as argued above, is *essential* to what I know in knowing (1). But only I can use that 'I' to index me—no being distinct from me can do so. I am not omniscient. But there is something that I know that no being distinct from me can know. Neither I nor any being distinct from me, then, is omniscient: there is no omniscient being.

A being distinct from me *could*, of course, know (3):

(3) Patrick Grim is making a mess.

But as argued above this does not amount to what I know in knowing (1). (Grim 1985: 154)

# References

# References

- Boethius (524 CE/2008) *The Consolations of Philosophy*, David R Slavitt, trans. Harvard University Press.
- Fitch, Frederic B (1963) 'A Logical Analysis of Some Value Concepts', *Journal of Symbolic Logic* **28**: 135–42. doi:[10.2307/2271594](https://doi.org/10.2307/2271594).
- Frankfurt, Harry G (1969) '[Alternate Possibilities and Moral Responsibility](#)', *Journal of Philosophy* **66**: 829–39.
- Goldman, Alvin I (1967) 'A Causal Theory of Knowing', *The Journal of Philosophy* **64**: 357–72. doi:[10.2307/2024268](https://doi.org/10.2307/2024268).
- Grim, Patrick (1985) 'Against Omniscience: The Case from Essential Indexicals', *Noûs* **19**: 151–80. doi:[10.2307/2214928](https://doi.org/10.2307/2214928).
- Grim, Patrick (1988) 'Logic and Limits of Knowledge and Truth', *Noûs* **22**: 341. doi:[10.2307/2215708](https://doi.org/10.2307/2215708).
- Grim, Patrick (2013) 'Problems with Omniscience', in J P Moreland, Chad Meister and Khaldoun A Sweis, eds., *Debating Christian Theism*: 169–80. Oxford University Press.
- Kaplan, David and Richard Montague (1960) 'A Paradox Regained', *Notre Dame Journal of Formal Logic* **1**: 79–90. doi:[10.1305/ndjfl/1093956549](https://doi.org/10.1305/ndjfl/1093956549).

## References (cont.)

- Leftow, Brian (1991) 'Timelessness and Foreknowledge', *Philosophical Studies* **63**: 309–25. doi:[10.1007/bf00354196](https://doi.org/10.1007/bf00354196).
- Lewis, David (1976) 'The Paradoxes of Time Travel', *American Philosophical Quarterly* **13**: 145–52.
- Lewis, David (1981) 'Are We Free to Break the Laws?', *Theoria* **47**: 113–21. doi:[10.1111/j.1755-2567.1981.tb00473.x](https://doi.org/10.1111/j.1755-2567.1981.tb00473.x).
- Oppy, Graham and Michael Scott, eds. (2010) *Reading Philosophy of Religion*. Wiley-Blackwell.
- Perry, John (1979) 'The Problem of the Essential Indexical', *Noûs* **13**: 3–21. doi:[10.2307/2214792](https://doi.org/10.2307/2214792).
- Pike, Nelson (1965) 'Divine Omniscience and Voluntary Action', *The Philosophical Review* **74**: 27–46. doi:[10.2307/2183529](https://doi.org/10.2307/2183529).
- Routley, Richard (2010) 'Necessary Limits to Knowledge: Unknowable Truths', *Synthese* **173**: 107–22. doi:[10.1007/s11229-009-9679-5](https://doi.org/10.1007/s11229-009-9679-5).
- Tymoczko, Thomas (1984) 'An Unsolved Puzzle about Knowledge', *The Philosophical Quarterly* **34**: 437–58. doi:[10.2307/2219063](https://doi.org/10.2307/2219063).

## References (cont.)

Zagzebski, Linda (2017) 'Foreknowledge and Free Will', in Edward N Zalta, ed., *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University.  
<https://plato.stanford.edu/archives/sum2017/entries/free-will-foreknowledge/>.